Decoupled Dark Matter

František Herman

October 22, 2014

Abstract

The WIMP[1] is probably the most popular dark matter particle candidate presented in the standard literature[**?**],[**?**]. The origin of their presence in the present Universe is explained by the process of **freezout** from the Local Thermal Equlibrium at the temperature scale $T \cong m_X/20$[**?**], where m_X stands for the WIMP's mass. Our main target is to explore possibility of the dark matter being made of **decoupled** particle species. We show that the estimation of the order of magnitude for the cross sections keeping these particles in the local thermal equilibrium is on the scale of the Weak interactions.

1 Main Idea

The main idea is very simple. Let us consider the traditional distribution function of the particles in the Local Thermal Equilibrium (LTE) with the energy ε, mass m and chemical potential μ:

$$f(\varepsilon) = \frac{1}{e^{\frac{\varepsilon-\mu}{T}} \pm 1}, \tag{1}$$

where upper sign holds for fermions and lower sign for bosons. Next, assume the density of states $\varphi(\varepsilon)$ of these particles with g internal degrees of freedom in the volume V in the isotropic case[**?**]:

$$\varphi(\varepsilon) = \frac{gV}{2\pi^2}\varepsilon\sqrt{\varepsilon^2 - m^2}, \tag{2}$$

where we consider $\varepsilon^2 = \mathbf{p}^2 + m^2$.

Now, let us to consider the decoupling process of the nonrelativistic particle species with the mass much greater than the decoupling temperature, $m \gg T_D$. We imagine the decoupling process as such abortion of the interactions of the considered particle species during which, the distribution function remains the same[2]. The distribution function of our hypotetic cold dark matter particles with the nonzero value of the chemical potential, marked with the Υ, after the decoupling reads:

$$f_\Upsilon(p) \cong \frac{1}{e^{\frac{p^2}{2mT_\Upsilon} + \frac{m-\mu}{T_\Upsilon}} \pm 1}. \tag{3}$$

[1]Weakly Interacting Massive Particle.

[2]Based on our opinion, this is the main difference between the idea of **decoupling** and **freezout**.

From this point we start to explicitly distinguish between the temperature of the decoupled DM particles T_Υ and the temperature T of the particle species remaining it the LTE. When we consider that the momentum of the Υ particles scales with the scaling parameter a as $p \propto a^{-1}$, we can easily compute the scaling of the temperature and the chemical potential for the considered decoupled DM particles[**?**]:

$$\begin{aligned} T_\Upsilon &= \frac{a_D^2}{a^2} T_D & \mu &= m - \left(\frac{a_D}{a}\right)^2 (m - \mu_D), \\ &= \frac{T^2}{T_D} & &= m - \left(\frac{T}{T_D}\right)^2 (m - \mu_D). \end{aligned} \tag{4}$$

The index D marks the value of the considered feature at the time of the decoupling. We have also used the fact that the scaling parameter a can be expressed throuht the photon temperature as $\frac{a_0}{a} = \frac{T}{T_0}$, and that the decoupling temperature of the Υ particles is equal to the temperature of the other species remaining in the LTE for the very last time. In fact, the introduced time dependence of the chemical potential keeps the value of the second fraction in the exponent in the eq.: (3) equal to its value at the decoupling temperature: $(m - \mu_D)/T_D = (m - \mu)/T_\Upsilon$. For further purposes we use the following notation:

$$\zeta \equiv \frac{m}{T_D}, \qquad \xi \equiv \frac{m - \mu_D}{T_D}. \tag{5}$$

Next, let us calculate the particle density and the energy density for the decoupled Υ particles using relations (2), (3) and integrals $I_a(\pm, \xi)$:

$$I_\pm(a,\xi) = \int_0^\infty \frac{x^a dx}{e^{x+\xi} \pm 1} \tag{6}$$

$$n_\Upsilon = \frac{1}{V}\int_0^\infty dp\varphi(p) f_\Upsilon(p) = \frac{g_\Upsilon \zeta^{\frac{3}{2}}}{\sqrt{2}\pi^2} I_\pm(1/2,\xi) T^3 \tag{7}$$

$$\rho_\Upsilon = \frac{1}{V}\int_0^\infty dp\varepsilon\varphi(p) f_\Upsilon(p) = \left(\zeta + \frac{I_\pm(3/2,\xi)}{I_\pm(1/2,\xi)}\left(\frac{T}{T_D}\right)^2\right) T_D n_\Upsilon \tag{8}$$

1.1 Consistency with the ΛCDM Cosmology

Assuming dark matter being made of the nonrelativistic decoupled particle species and considering its contribution to the energy content of the Universe at the early times $T \lesssim 1TeV$, we can obtain condition on the allowed values for ξ.

Indroducing individual contributions to the energy density at the early Universe from the radiation, baryonic matter, nondecoupled dark matter and the decoupled dark matter particles, we are left with:

$$\rho = \rho_{\gamma 0}\left(\frac{T}{T_0}\right)^4 + \underbrace{(\rho_{m0} - \rho_{\Upsilon 0})\left(\frac{T}{T_0}\right)^3 + \rho_\Upsilon}_{\rho_m}, \tag{9}$$

We are going to assume, that the whole dark matter content of our observable Universe is made of decoupled Υ particles. All of the presented calculations can be easily generalized in the situations, when $\Omega_{\Upsilon} < \Omega_{dm}$. Examining an eq.(9) together with an eq.(8) we can see that for the Υ particles we count also the kinetic energy of these particles. However, for the other massive particle species contributing to the matter content, we count only the rest energy of these particles. The motivation for this step is following. The contributions from the kinetic energy of the baryons and alternatively standard, frozen out dark matter would be small against the energy density of the relativistic species. However, the contribution from the kinetic energy of the decoupled Υ particles could be bigger than the energy density of the remaining relativistic particle species, since it depends on the value of the mass, chemical potential and decoupling temperature.

We can also represent this idea graphically. At first we can easily rearrange eq.(9), using presented equations for the particle density eq.(7) and the energy density eq.(8) of the decoupled dark matter, in to the form:

$$\frac{\rho}{T^4} = \underbrace{\left(\frac{(\rho_{m0} - \rho_{\Upsilon 0})}{T_0^4} \delta + \frac{g_{\Upsilon} \zeta^{\frac{5}{2}}}{\sqrt{2}\pi^2} I_{\pm}(1/2, \xi) \right) x + \frac{g_{\Upsilon} \zeta^{\frac{3}{2}}}{\sqrt{2}\pi^2} I_{\pm}(3/2, \xi) x^{-1}}_{\rho_m/T^4} + \underbrace{g_*(T) \frac{\pi^2}{30}}_{\rho_{\gamma}/T^4}, \tag{10}$$

where we use following motation: $\delta = \frac{T_D}{T_0}$ and $x = \frac{T_D}{T}$. For some values of ξ, after the decoupling of the Υ particles, the energy density would be dominated by the term proportional to $(a_0/a)^5$ (kinetic energy density of Υ particles) and after that by the term prop. to $(a_0/a)^3$ (rest energy density of the all matter particles). Since the results of observations agree with the ideas of the ΛCDM cosmology, which determines the value of the photon temperature at the time of equality between the matter and radiation: $T_{eq} = \Omega_m/\Omega_{\gamma} T_0$, we constrict ourselves on the such values of ξ, which allow the existence of the radiation domination era, even after the decoupling of the Υ particles: At first, we start from the condition on the existence of the equality era, $\rho_{\gamma}(T_{eq}) = \rho_m(T_{eq})$, which can be rearranged by using eq.(7),(8):

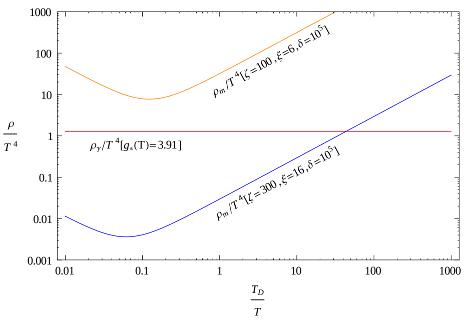


Figure 1: Ilustration of the existance and non-existance of the equality era between radiation and matter energy densities in the model with the decoupled Υ dark matter particles, which depends on the values of ζ and ξ.

$$\left(\frac{T_D}{T_{eq}}\right)^2 \frac{g_{\Upsilon} \zeta^{\frac{5}{2}}}{\sqrt{2}\pi^2} I_{\pm}(1/2, \xi) - \frac{T_D}{T_{eq}} g_*(T_0) \frac{\pi^2}{30} \frac{\Omega_{\Upsilon}}{\Omega_m} + \frac{g_{\Upsilon} \zeta^{\frac{3}{2}}}{\sqrt{2}\pi^2} I_{\pm}(3/2, \xi) = 0 \tag{11}$$

Constraining condition in the implicit form on the allowed values of ξ can be easily found as the positive value of the discriminant for the constructed quadratic equation eq.(11). In addition, requirement that our results have to agree with the benchmark model of the ΛCDM Cosmology, the energy density term in the translated form: $\Omega_{\Upsilon kin.}\left(\frac{a_0}{a}\right)^5$, which comes from the contribution of the kinetic energy density of Υ particles should be negligible. In such a case, our condition for the allowed values of ξ and ζ obtains a form:

$$\left(\frac{g_*(T_0)}{g_\Upsilon}\frac{\pi^4}{30\sqrt{2}}\frac{\Omega_\Upsilon}{\Omega_m}\right)^2 \gg \zeta^4 I_\pm(3/2,\xi)I_\pm(1/2,\xi) \tag{12}$$

Realizing that the number on the LHS is just the number from the interval (1,10), next that $\zeta \gg 1$ and the form of the integrals $I_\pm(a,\xi)$, eq.(6), we can assume (and the numerical analysis will prove us right) that the ξ should be positive number. Numerical analysis shows that the interval for the allowed values of ξ is actually: $\xi \gtrsim 6$. For these values of ξ, we can safely use the classical limit for the integrals $I_\pm(a,\xi)$, where the difference between the boson and fermion species is wiped out:

$$I_\pm(a,\xi) = \Gamma(a+1)e^{-\xi}. \tag{13}$$

Our constrain in the form of eq.(12) for the allowed values for ξ and ζ obtains a simple form:

$$-1.84 + log_{10}\sqrt{\frac{g_*(T_0)}{g_\Upsilon}\frac{\Omega_\Upsilon}{\Omega_m}} + 0.22\xi \geq log_{10}\zeta \tag{14}$$

1.2 Decoupled Cold Dark Matter Antiparticles

Let us consider the particle density of the decoupled Υ antiparticles, assuming conditions presented in the previous section. Comparing it with the eq.(7) the only change would be in the sign of the chemical potential. Since we assume that the total number of the Υ particles and antiparticles remains fixed after their decouplement, it can be shown[?] that the chemical potential of the Υ antiparticles would be the minus chemical potential for the Υ particles. Considering this, we obtain:

$$n_{\bar{\Upsilon}} = n_\Upsilon e^{2\left(\xi-\zeta\left(\frac{T_D}{T}\right)^2\right)} \tag{15}$$

As we can see their particle density would be suppressed by the factor $e^{\left[2\left(\xi-\zeta\left(\frac{T_D}{T}\right)^2\right)\right]}$ against the particle density of the Υ particles. At the first sight, this fact causes trouble, because it would mean that the comoving density of the dark matter antiparticles does not remain constant after the decoupling. It would mean that even when the Υ particle species decouple, the amount of its antiparticles continue to decrease.

However, it shows that there is in fact no problem at all. When we assume that the total number of Υ particles and antiparticles is fixed even before decoupling[3], the particle density for the dark matter antiparticles in the LTE would be described by the eq.(15). Naturally, we must keep on our minds, that the presented equation remains true only in the case of the homogeneous and isotropic distributed dark matter in the Universe. Since the homogeneous part

[3] "Dark matter particle number conservation"

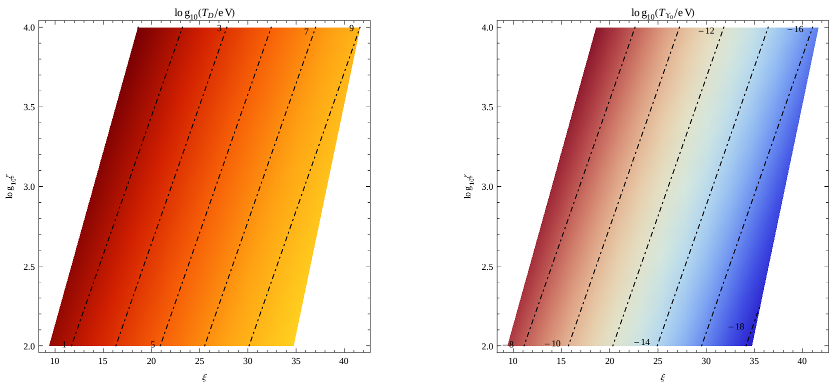
of the particle density for the antiparticles is exponentially surpressed during the LTE, we should also account the inhomogeneous corrections. Without the corrections the eq.(15) is incomplete. Considering the presented idea, we conclude that for the proper examination of the comoving particle density of the decoupled Υ antiparticles, we would have to include influence of inhomogeneities.

1.3 Temperature, Particle Density and Mean Velocity for Decoupled Υ Particles

Once we had ensured that there will be also dominance of the radiation component in the evolution of the Universe in the model with the decoupled dark matter particles, we can neglect the third term in the eq.(11), since the terms representing contribution from the rest energy density of the Υ particles (first term) and the contribution from the mentioned energy density of the radiation component (second term) are dominant. Solving the eq.(11) and using known value for the $T_{eq} = \Omega_m/\Omega_\gamma T_0$ obtained from the ΛCDM Cosmology itself, we find relation between the decoupling temperature of the Υ particles and their mass and chemical potential at the time of the decoupling:

$$T_D = 5.18 \frac{g_*(T_0)}{g_\Upsilon} \frac{\Omega_\Upsilon}{\Omega_m} \frac{e^\xi}{\zeta^{\frac{5}{2}}} \tag{16}$$

As we can see, the decoupling temperature increases exponentially with the ξ and decreases with the $5/2$ power of ζ. It also linearly increases with the ratio of the amount of the matter component being created by the decoupled Υ particles to the whole amount of the matter component. For better illustration, we plot



(a) (b)

Figure 2: Contour plots of the decoupling and present temperatures for the Υ particles.

the contour plot of T_D in the fig.2a[4]. We also plot the present temperature of the Υ particles obtained by using eq.4 in the fig.2b. We can immidiatelly notice that the present temperature of the earlier decoupled Υ particles will be lower

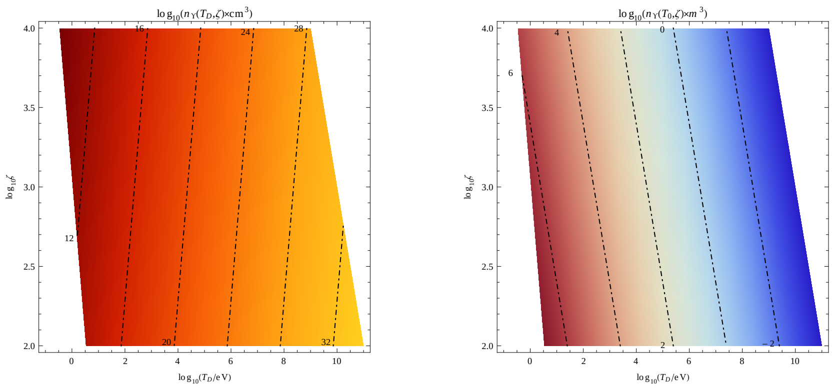
[4]Restrictions on the plotted regions come from the condition eq.14 and from the maximal energy per particle being less than the tens of TeV.

than the temperature of the Υ particles which have decoupled later after the Big Bang. This effect is caused by the longer time which these earlier decoupled particles spend outside the Local Thermal Equilibrium.

Using e.g. eq.(7) and eq.(16) we can also easily find the equation for the particle density at the decoupling temperature:

$$n_\Upsilon(T_D) = 0.33 g_*(T_0) T_{eq} \frac{\Omega_\Upsilon}{\Omega_m} \frac{T_D^2}{\zeta} \tag{17}$$

The contour plot for the particle density at the decoupling time is presented in the fig.3a. Realizing that $n_\Upsilon(T_0) = n_\Upsilon(T_D)\,(T_0/T_D)^3$, we can also easily construct the contour plot for the present particle density of the Υ particles, presented in the fig. 3b. Let us assume two different kinds of Υ particles, both



(a) (b)

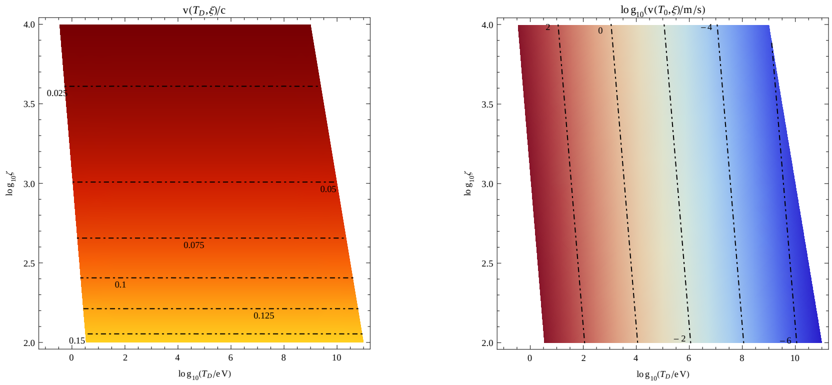
Figure 3: Contour plots of the decoupling and present particle densities for the Υ particles.

with the same ζ: ratio of their mass and the decoupling temperature. Once again, in the fig.3a, we can see that the particle density at the decoupling temperature for the Υ particles which decoupled at the higher temperature scale is higher than the particle density of the particles which remain in the LTE longer time and decoupled at the lower temperature scale. On the other hand as we can see in the fig.3b, the longer period of time which the heavier particles have spent outside the LTE will reduce their particle density way bellow the particle density of the lighter particles which would decouple later.

To complete our collection of properties for the considered decoupled Υ dark matter particle species, we also present their mean velocities at the decoupling (fig.4a) and present (fig.4b) temperatures. The mean velocity is obtained by computing:

$$v_\Upsilon = \frac{1}{n_\Upsilon V} \int_0^\infty dp \frac{p}{m} \varphi(p) f_\Upsilon(p) = \frac{1.6}{\sqrt{\zeta}} \frac{T}{T_D} \tag{18}$$

The basic idea of all of the presented plots in here is to scatch the basic concept of crucial properties for our considered decoupled particle species.



(a) (b)

Figure 4: Contour plots of the decoupling and present mean velocities for the Υ particles.

1.4 Estimation of Cross Sections at Decoupling Temperature Scale

The basic idea of the presented estimation of the order of magnitude for the cross sections of the interactions that keep Υ particles in the contact with the LTE for the very last time is following: We are going to compare characteristic time between the collisions τ which keep the Υ particles in the LTE with the Hubble time t_H, which represents the age of the observable Universe at the considered temperature scale. It should be intuitively clear that in the regime: $\tau \ll t_H$, the equilbrating interactions are very frequent, whereas when the $\tau \gg t_H$ the particle does not interact at all and remains absolutelly decoupled from the other species which create the LTE. Considering these two mentioned limit regimes, we will consider condition for the occurence of the decouplement in the form: $\tau(T_D) = t_H(T_D)$.

At first we will have a look at the Hubble time. Since we are interested in the Universe where the most important contributions to the energy density come from the radiation and matter, the Friedmann equation obtains a form:

$$H^2 = H_0^2 \left(\Omega_\gamma \left(\frac{a_0}{a}\right)^4 + \Omega_m \left(\frac{a_0}{a}\right)^3 \right). \tag{19}$$

Using eq.19 for the calculation of the Hubble time at the temperature of the decoupling $t_H(T_D) = t_D$, we are left with:

$$\begin{aligned} t_D &= \int_0^{a_D} \frac{da}{aH_0\sqrt{\Omega_\gamma \left(\frac{a_0}{a}\right)^4 + \Omega_m \left(\frac{a_0}{a}\right)^3}} \\ &= \frac{2}{H_0\sqrt{\Omega_m}} \left(\frac{T_0}{T_{eq}}\right)^{\frac{3}{2}} \left(\frac{2}{3} + \frac{1}{3}\left(1 + \frac{T_{eq}}{T_D}\right)^{\frac{3}{2}} - \left(1 + \frac{T_{eq}}{T_D}\right)^{\frac{1}{2}} \right) \end{aligned} \tag{20}$$

Now, let us describe the characterictic time between collisions, τ. We understand it as 1/Collision rate, where the Collision rate $\equiv \Gamma(T)$ measures the number of equilbrating interactions over the time in the system with the temperature T. The collision rate Γ depends on the weighted cross section multiplied by the relative velocity of the interacting particles $\langle \sigma v_{rel.} \rangle$ and also on the particle density of the target particles $n_{tar.}(T)$ within the temperature T on which the equilbrating processes occure. In principle, there could be two different kinds of equilbrating processes contributing to the Γ. The first one will measure the contribution from the interactions between the particles Υ and other particles (relativistic species marked as γ, since the standard nonrelativistic species have very small particle densities):

$$\Gamma_{\gamma\Upsilon} = \langle \sigma_{\gamma\Upsilon} v_{rel\gamma\Upsilon} \rangle n_{\gamma}. \tag{21}$$

The second term corrensponds to the contribution from the equilbrating processes which run mostly between the Υ particles themselves.

$$\Gamma_{\Upsilon\Upsilon} = \langle \sigma_{\Upsilon\Upsilon} v_{rel\Upsilon\Upsilon} \rangle n_{\Upsilon}. \tag{22}$$

Considering these two parts of the collision rate, our condition for the occurrence of the decoupling obtains a form:

$$\Gamma_{\gamma\Upsilon}(T_D) + \Gamma_{\Upsilon\Upsilon}(T_D) = \frac{1}{t_D(T_D)} \tag{23}$$

If the dark matter is really created by particles then we should obtain the introduced cross sections from some model of the quantum field theory, describeing the characterictics of the interactions for these dark matter particles with other kinds of particles and also with themselves. It may happen that the difference between the coupling constants which would multiply the corresponding interaction terms for the important equilbrating processes would be several orders of magnitude. Such a difference between the coupling constants would lead to the differences in the values of the cross sections describing the processes, important for the equalibration of our considered decoupled Υ particles. We use this idea in the following estimation of the order of magnitude for the cross sections corresponding to equilbrating processes at the era of Υ decoupling.

At first, we assume that $\Gamma_{\Upsilon\Upsilon} \gg \Gamma_{\gamma\Upsilon}$, which would mean:

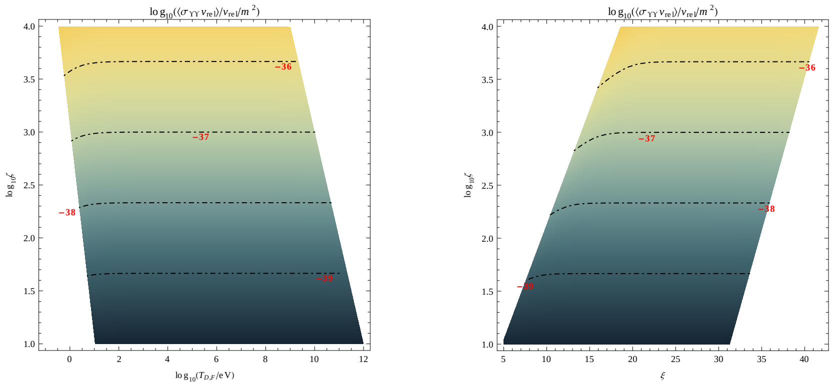
$$\frac{\langle \sigma_{\Upsilon\Upsilon} v_{rel\Upsilon\Upsilon} \rangle}{\langle \sigma_{\gamma\Upsilon} v_{rel\gamma\Upsilon} \rangle} \gg \frac{n_{\gamma}}{n_{\Upsilon}}. \tag{24}$$

Realizing that the right hand side of this condition varies from the $10^3 - 10^{12}$ based on the mass and decoupling temperature of our considered Υ particle species, we assume huge difference in the interactions of Υ particles with the other particles species and the interactions among themselves. However, if this assumption is satisfied, we can neglect the first term in the condition for the Υ decoupling. Now, we estimate the order of magnitude for the cross section in the form: $\langle \sigma_{\Upsilon\Upsilon} v_{rel\Upsilon\Upsilon} \rangle / v_{rel\Upsilon\Upsilon}$, where the relative velocity for the Υ particles can be in our case $\xi \gtrsim 6$ easily calculated:

$$
\begin{aligned}
v_{rel\Upsilon\Upsilon} &= \frac{g_\Upsilon^2}{n_{\Upsilon_1} n_{\Upsilon_1} m} \int_0^\infty \frac{dp_1}{2\pi^2} p_1^2 f_\Upsilon(p_1) \int_0^\infty \frac{dp_2}{2\pi^2} p_2^2 f_\Upsilon(p_2) |\mathbf{p_1} - \mathbf{p_1}| \\
&= 2 v_\Upsilon,
\end{aligned} \tag{25}
$$

where the v_Υ si well known from the eq.18. Putting everything together, we are left with:

$$
\frac{\langle \sigma_{\Upsilon\Upsilon} v_{rel\Upsilon\Upsilon} \rangle}{v_{rel\Upsilon\Upsilon}} = \frac{0.12 \left(\frac{\sqrt{\Omega_\gamma \zeta}}{T_0} \right)^3 \frac{H_0}{\Omega_\Upsilon}}{\left(\frac{T_D}{T_{eq}} \right)^2 \left(\frac{2}{3} + \frac{1}{3} \left(1 + \frac{T_{eq}}{T_D} \right)^{\frac{3}{2}} - \left(1 + \frac{T_{eq}}{T_D} \right)^{\frac{1}{2}} \right)} \tag{26}
$$



(a) (b)

Figure 5: Properties of the decoupling Υ dark matter particles on the different temperature scales in the limit: $\Gamma_{\Upsilon\Upsilon} \gg \Gamma_{\gamma\Upsilon}$

Obtained results are plotted in the fig.5a and 5b. The plot presented in the fig.5a should be read as the answer to the following question: What are the properties of the particle species Υ, which would decouple on the considered temperature scale? As we can see, the estimation for the equilbrating cross section shows the scale of the Weak interactions for the decoupled particles, which mass is approx. $10 - 100$ times larger then the temperature scale at the time of their decouplement.

Since the contours of the decoupling cross sections are nearly straight lines[5], this statement remains true for the most of the considered temperature scales. We can connect this new piece of information together with the fig.3a which presents the particle density at the decoupling temperature. Let us consider some temperature scale. As we can see from the fig.5a on this scale would decouple particle species with higher mass and higher cross section, or particles with lower mass and lower cross section. Examining the fig.3a and the fig.4a we see that the heavier particles would have lower particle density and lower

[5]The small bend is caused by the starting matter dominance at the temperature scale $T \lesssim 1eV$.

relative velocity and viceverse, the lighter particles would have higher particle density and higher relative velocity instead.

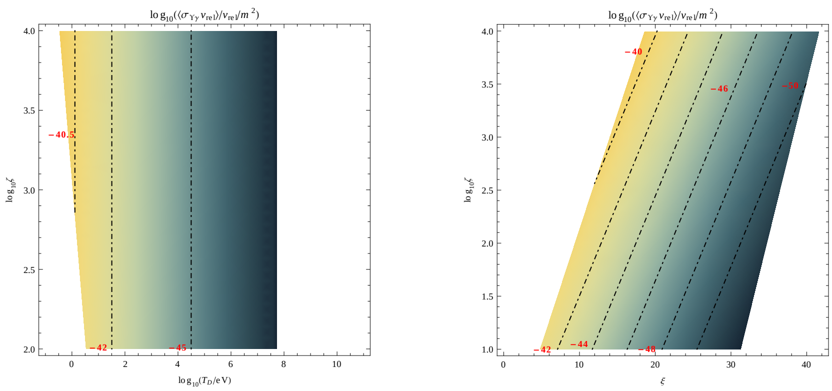
The obtained result could be also obtained by absolutelly different idea. Because the important role in our cross section estimation plays the particle density of the dm particles and not the distribution function itself, the result could be obtained also by taking the present particle density of the dm particles (obtained from the known energy density for the dark matter), which will of course depend from the mass of dm particles, then express it at the temperature when the particles left from the LTE. We purposely do not specify the way how do these particles left the LTE, since all we have used is the particle density and not the distribution function. The results of the estimation for the cross section using these ideas would be the same as the one presented in the fig.5a At the end, if we would use the result from the solution of the Boltzmann equation for the freezout of the dark matter particles: $\zeta \simeq 20$, we can see that we would obtain just the Weak interaction scale from the cross section estimation well known from the Wimp miracle[**?**].

Next, we are going to examine the opposite limit, $\Gamma_{\Upsilon\Upsilon} \ll \Gamma_{\gamma\Upsilon}$ in which we assume:

$$\frac{\langle \sigma_{\gamma\Upsilon} v_{rel\gamma\Upsilon} \rangle}{\langle \sigma_{\Upsilon\Upsilon} v_{rel\Upsilon\Upsilon} \rangle} \gg \frac{n_\Upsilon}{n_\gamma}. \tag{27}$$

The assumption which we use in this case is bassically opposite to one which we presented in the previous paragraph, working in the regime, where was satisfied condition eq.12. Realizing that the relative velocity $v_{rel.\Upsilon\gamma} \cong 1$, (in the units where $c = 1$), the final result is:

$$\langle \sigma_{\gamma\Upsilon} \rangle = \frac{\frac{1.05}{g_*(T_D)} \frac{H_0 \Omega_m^{\frac{1}{2}}}{(T_0 T_{eq})^{\frac{3}{2}}}}{\left(\frac{T_D}{T_{eq}}\right)^3 \left(\frac{2}{3} + \frac{1}{3}\left(1 + \frac{T_{eq}}{T_D}\right)^{\frac{3}{2}} - \left(1 + \frac{T_{eq}}{T_D}\right)^{\frac{1}{2}}\right)} \tag{28}$$



(a) (b)

Figure 6: Properties of the decoupling Υ dark matter particles on the different temperature scales in the limit: $\Gamma_{\Upsilon\Upsilon} \ll \Gamma_{\gamma\Upsilon}$

Our obtained results are ploted in the fig.6a and fig.6b. As we can see from the eq.27 and from the fig.6a the decoupling temperature scale is in this considered limit not dependent on the mass of the dm particles. As mentioned earlier the fig.6a can be used not just for the decoupled particles, but also for particle species which simply freezed out from the LTE at the temperature scale $T_{D,F}$. However the plot fig.6b shows the estimation for the cross sections at the decoupling temperature scale for decoupled particles with different values of ζ and ξ.

As we can see the estimated decoupling cross sections for the Υ particles which would be kept in the thermal equilibrium mostly by interactions with the other relativistic particle species would be once again on the scale of the Weak interactions. Once again the plot in the fig.6a can be also used as the freezout scale for the dm particles.

It is good to realize, that the estimation for the value of the cross section at the decoupling temperature in the limit $\Gamma_{\Upsilon\Upsilon} \ll \Gamma_{\gamma\Upsilon}$ presented in the fig.6a remains absolutelly the same even for light, relativistic species dropping out of the LTE, since the only important quantities are: t_H, $v_{rel.}$ and the particle density of the taget particles: n_γ.

1.5 Aknowledgement

Dosiahnuté výsledky pri riešení problému asymetrickej tmavej látky s nenulovým chemickým potenciálom boli dosiahnuté s podporou Ministerstva školstva, vedy výskumu a športu SR v rámci poskytnutia dotácie v zmysle § 8a zákona č.172/2005 Z. z. o organizácii štátnej podpory výskumu a vývoja a o doplnení zákona č. 575/2001 Z. z. o organizácii činnosti vlády a organizácii ústrednej štátnej správy v znení neskorších predpisov v platnom znení.

References